Development of Attribute Preserving Network Equivalents

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Acknowledgements

Overview

- Objective - develop attribute preserving power system network equivalents
  - preserve the essence of a model for some purpose
- Desirable properties include…
  - Economic analysis of electric power systems including transfer capacity
  - Transient stability response
  - LMP characteristics
  - **Application Dependent!!**
- Present focus is on developing equivalents that preserve the line limits of the original system

Preserving Transmission Line Limits

- For decades power system network models have been equivalenced using the approach originally presented by J.B. Ward in 1949 AIEE paper "Equivalent Circuits for Power-Flow Studies"
  - Paper's single reference is to 1939 book by Gabriel Kron, so this is also known as Kron's reduction
  - Certainly other techniques, but Ward's is common
- System buses are partitioned into a study system (s) to be retained and an external system (e) to be eliminated; buses in study system that connect to the external buses are known as boundary buses

Ward Equivalents

- No impact on study, non-boundary buses

$$\begin{bmatrix} I_s \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ss} & Y_{se} \\ Y_{es} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_s \\ V_e \end{bmatrix}$$

$$\left(I_s - Y_{se} Y_{ee}^{-1} I_e \right) = \left(Y_{ss} - Y_{se} Y_{ee}^{-1} Y_{es} \right) V_s$$

- Equivalent is created by doing a partial factorization of the Ybus
  - Computationally efficient

Equivalencing Process

- As each equivalent bus is conceptually removed during the partial factorization, equivalent admittance is added to the bus admittance matrix
  - In essence setting up an equivalent line between the buses directly connected to the bus being equivalenced
- This is an iterative process that occurs as each bus is equivalenced. So as bus k is equivalenced, connected to retained buses i and j, then

$$Y_{ij}' = Y_{ij} - \frac{Y_{ik}Y_{kj}}{Y_{kk}}$$

First term is admittance of any existing line (which may be zero), second term is for the new, equivalent line

Equivalencing Process

- Power injections at equivalent buses (e.g., from generation and load) are assigned to the retained buses

- At end of process the equivalent admittance values are converted into equivalent lines, with resistance and reactance values

- Boundary bus connections can be quite dense, so usually high impedance equivalence lines are removed (ignored)

Common Equivalencing

- Often the study system corresponds to a utility or ISO and its near neighbor buses
  - A compact, contiguous system
- The external system is the rest of the interconnect
  - Usually not highly concerned about the external system
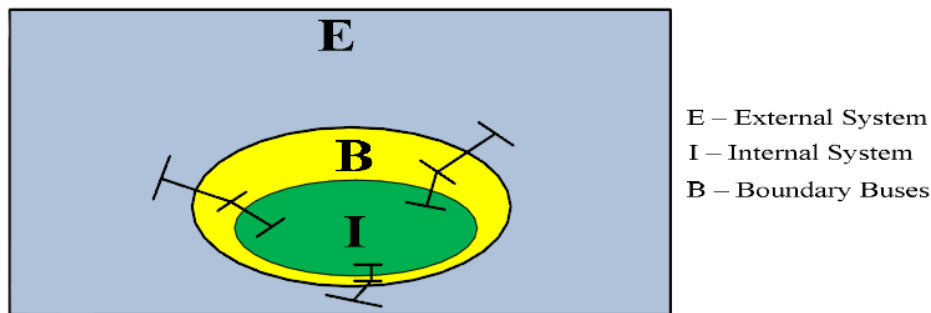 - In an EMS limited real-time information would be available for the external system


E – External System
I – Internal System
B – Boundary Buses

Image: Fig. 1 from D.Shi, D.L. Shawhan, N. Li, D.J. Tylavsky, J.T. Taber, R.D. Zimmerman, W.D. Schulze, "Optimal Generation Investment Planning: Pt. 1: Network Equivalents, Proc. 2012 NAPS, Champaign, IL Sept 2012

Backbone Type Equivalents

- A newer equivalent is one in which the whole geographic extent of the network is retained, but with potentially a greatly reduced bus count
  - Might be used in longer term economic studies
  - Study area is not localized but spread across the system
  - Key benefit is potential for much faster computations

E – External System
I – Internal System
B – Boundary Buses

Image: Fig. 2 from D.Shi, D.L. Shawhan, N. Li, D.J. Tylavsky, J.T. Taber, R.D. Zimmerman, W.D. Schulze, "Optimal Generation Investment Planning: Pt. 1: Network Equivalents, Proc. 2012 NAPS, Champaign, IL Sept 2012
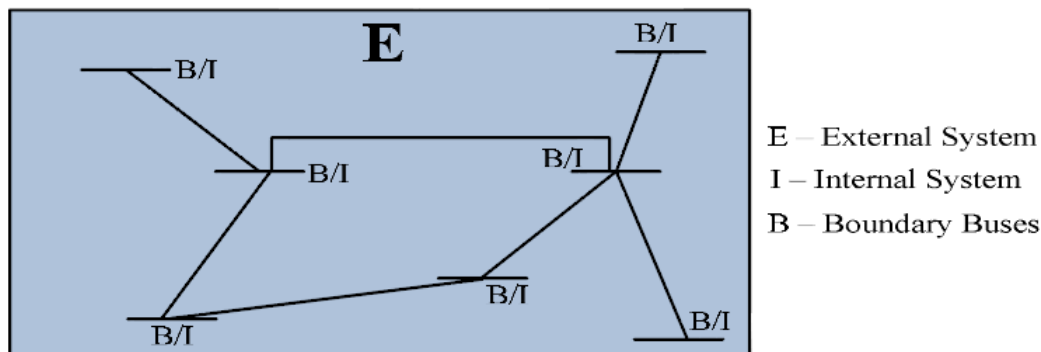
Retaining Equivalent Line Limits

- Existing Ward equivalencing algorithms do not assign limits to the equivalent lines
  - Essentially this means the line is assumed to have infinite MVA capacity
- Research presented here seeks algorithms to assign at least approximate limits to the new equivalent lines
  - Something better than infinite capacity!
- Information is lost during equivalencing, and the behavior of the equivalent will not (in general) match the unreduced system

Retaining Equivalent Line Limits

- Gist of our approach is to sequentially assign limits to these equivalent lines that preserve some desired attribute of the original network

- One attribute is total transfer capability (TTC) between the retained buses

- This is done on a bus by bus basis during the equvialencing process

Desired Characteristic:
Bus to Bus Transfer Capacity

- Desire is to have the TTC between retained buses match that for these buses in the original system

- Value is determined from PTDFs, ignoring loading

$$. = \min\left(\frac{\text{Line Limit}_l}{\left| \text{PTDF}_{ij,l} \right|} \right) \text{ for } l = 1, 2 .. L$$

- Idea is to do this sequentially, for all the first neighbors of bus k, as it is being eliminated

PTDF Characteristics

- The lossless (dc power flow) PTDFs can be determined from a factored B matrix using a fast-forward, full-backward substitution

  - Fast-backward can be used if just a few are needed as will be the case here

- PTDFs on study system lines are not affected by the equivalencing process, and those on the new equivalent lines can be easily calculated from the original system

Algorithm Overview

- Sequentially for each bus being equivalenced
  1. Calculate the PTDFs between the adjacent buses
  2. Using these PTDFs, determine the TTCs between the adjacent buses, just considering the limits on the lines that are being removed
     - Limits on the other lines do not need to be considered since these lines are being retained (at least until the next bus is considered).
  3. Select limits for the new equivalent lines so that the TTCs of the reduced system match that of the original system.
  4. Combine limits/impedances on parallel, equivalent lines

Combining Parallel Lines

- One consequence of this algorithm is the creation of lots of parallel, equivalent lines. Parallel line equivalencing is trivial, with the new limit just determined by determining which line in the parallel bundle is binding. That is, for n lines in parallel, each with impedance Z_i and total impedance Z_{combo}

$$Lim_{combo} = min\left(Lim_1 \frac{Z_1}{Z_{combo}}, Lim_2 \frac{Z_2}{Z_{combo}}, ..., Lim_n \frac{Z_n}{Z_{combo}}\right)$$
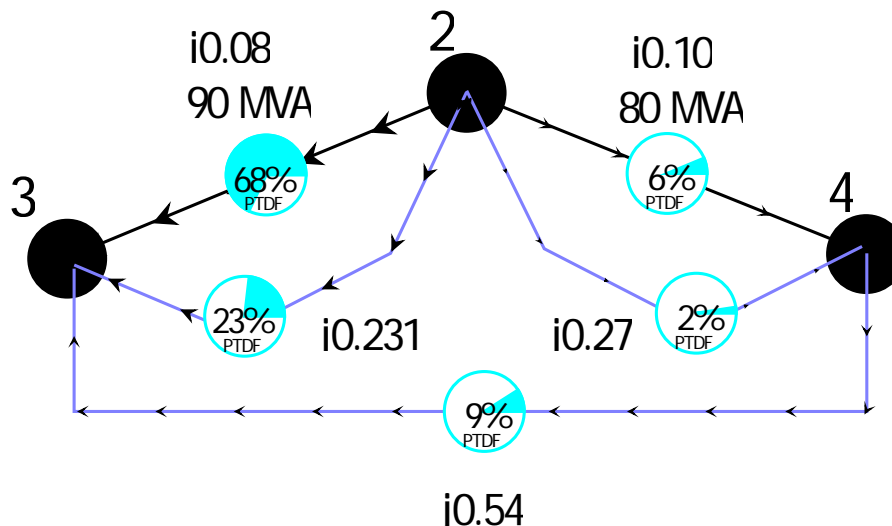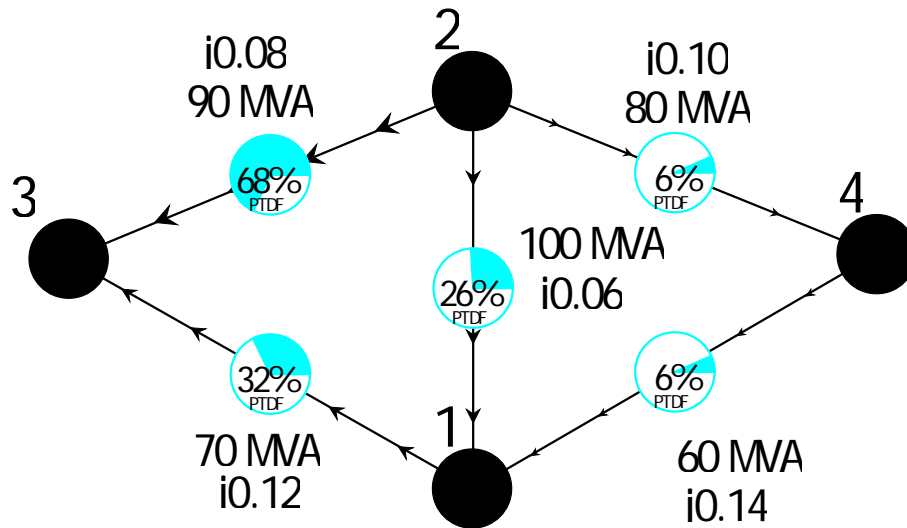
With removing bus 1, three equivalent lines will be added between the other three buses. The original TTCs are
2-3: 216.7 MW (1-3 binding)
2-4: 171.7 MW (1-4 binding)
3-4: 144.9 MW (1-4 binding)

For 2-3 direction for new equivalent line limits we require
1) Lim23 >= 216.7*0.234 = 50.7MW
2) Lim24 >= 216.7*0.024 = 5.2MW
3) Lim34 >= 216.7*0.088 = 19.1MW

Similar constraints for the other directions

Equality and Inequality Constraints

- At each iteration the limits for the equivalent lines must be set high enough that they do not reduce the TTCs in the original network
    - Inequality constraints
    - These inequality constraints occur for all new equivalent lines for each direction; with m adjacent buses there are $m*(m$-1)/2 directions and hence new lines
        - m = 3 in the example and hence 3*2/2 = 3 directions & new lines
- Limits must be set so there is a binding equality constraint in each direction

Four Bus Example, cont.

- To determine the new line limits we need to satisfy inequality and equality constraints

For the 2-3 direction for the new equivalent line limits we require
1) Lim23 >= 216.7*0.234 = 50.7 MW
2) Lim24 >= 216.7*0.024 = 5.2 MW
3) Lim34 >= 216.7*0.088 = 19.1 MW
And one must be an equality!

For the 3-4 direction for the new equivalent line limits we require
1) Lim23 >= 144.9*0.206 = 29.8 MW
2) Lim24 >= 144.9*0.217 = 31.4 MW
3) Lim34 >= 144.9*0.197 = 28.5 MW
And one must be an equality!

For the 2-4 direction for the new equivalent line limits we require
1) Lim23 >= 171.7*0.028 = 4.8 MW
2) Lim24 >= 171.7*0.241 = 41.4 MW
3) Lim34 >= 171.7*0.109 = 18.7 MW
And one must be an equality!

Often times the solution will be trivial, just picking the largest in each row. Here the answer is Lim23=50.7 MW, Lim24=41.4 MW and Lim34=28.5 MW

Matrix View

- Define a matrix view, with entries showing PTDFs x TTC; hence they give the minimum limit needed to allow for the original TTCs

- Require the largest in each row, and at least one entry per column (to bind the direction)

Directions

	2-3	2-4	3-4
Eqv Line 2-3	**50.7 MW**	4.8 MW	29.8 MW
Eqv Line 2-4	5.2 MW	**41.4 MW**	31.4 MW
Eqv Line 3-4	19.1 MW	18.7 MW	**28.5 MW**

Often the solution is just the diagonal entries

Complications!

- Sometimes an exact solution is not possible
- That is we can't satisfy all the inequality constraints, while also binding
- Solution is to bound the limits, or make a best guess
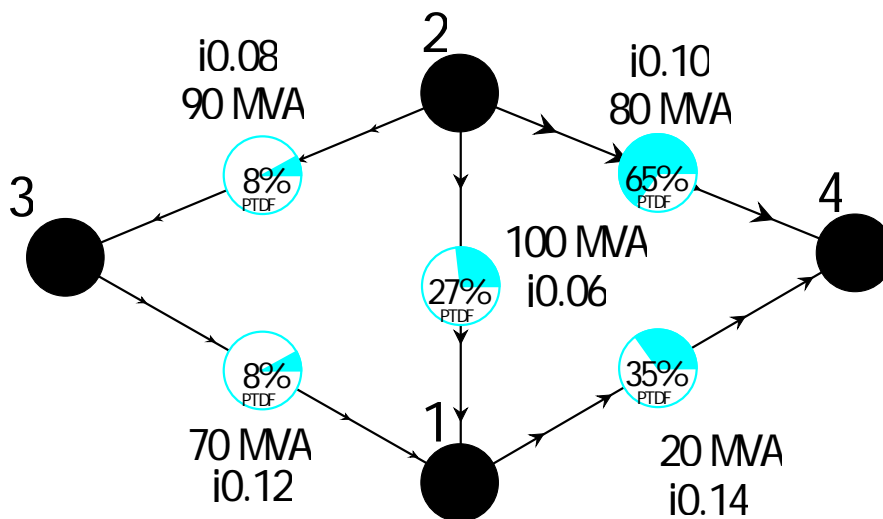
Directions (Original System Data)

	2-3	2-4	3-4
Eqv Line 2-3	**50.7 MW**	4.8 MW	29.8 MW
Eqv Line 2-4	5.2 MW	**41.4 MW**	31.4 MW
Eqv Line 3-4	19.1 MW	18.7 MW	**28.5 MW**

No Solution Example

- Original four bus case, except the limit on line 1-4 has been reduced to 20 MVA

Figure shows PTDFs from 2 to 4

TTCs using reduced limit

2-3: 216.7 MW (1-3 binding)
2-4: 57.2 MW (1-4 binding)
3-4: 48.3 MW (1-4 binding)

No Solution Example

- ## New limits that need to be satisfied

For the 2-3 direction for the new equivalent line limits we require
1) Lim23 >= 216.7*0.234 = 50.7 MW
2) Lim24 >= 216.7*0.024 = 5.2 MW
3) Lim34 >= 216.7*0.088 =19.1 MW
And one must be an equality!

For the 3-4 direction for the new equivalent line limits we require
1) Lim23 >= 48.3*0.206 = 9.9 MW
2) Lim24 >= 48.3*0.217 = 10.5 MW
3) Lim34 >= 48.3*0.197 = 9.5 MW
And one must be an equality!

For the 2-4 direction for the new equivalent line limits we require
1) Lim23 >= 57.2*0.028 = 1.6 MW
2) Lim24 >= 57.2*0.241 = 13.8 MW
3) Lim34 >= 57.2*0.109 = 6.2 MW
And one must be an equality!

All cannot be true simultaneously!

No Solution Example:
First Approach, Overestimate

- In the first approach we satisfy all of the inequality constraints. But here this means one of the equality constraints is not satisfied

Directions (Modified System Data)

	2-3	2-4	3-4
Eqv Line 2-3	**50.7 MW**	1.6 MW	9.9 MW
Eqv Line 2-4	5.2 MW	**13.8 MW**	10.5 MW
Eqv Line 3-4	**19.1 MW**	6.2 MW	9.5 MW

Allowable flow in direction 3-4 is overestimated since none of the entries in its column are enforced. Overestimated flow is 13.8/10.5 = 131% of the actual value.

General Solution Procedure, cont.

- In the second approach we insure all the equality constraints are satisfied, which insures that the flow in every direction is no more than its original TTC. But because some of the inequality constraints would be in violation, these limits under-estimate the TTC in at least some directions

- Solution is motivated by defining a "limit violation cost" for each matrix entry, which is the sum of violations for all entries in the row (other norms could be used!)

General Solution Procedure: Limit Violation Cost

	2-3	2-4	3-4
Eqv Line 2-3	**50.7 MW**	1.6 MW	9.9 MW
Eqv Line 2-4	5.2 MW	**13.8 MW**	10.5 MW
Eqv Line 3-4	**19.1 MW**	6.2 MW	9.5 MW

Directions: Limit Violation Costs

	2-3	2-4	3-4
Eqv Line 2-3	0	57.4	40.8
Eqv Line 2-4	13.9	0	3.3
Eqv Line 3-4	0	16.2	9.6

Example: For the first row, the 2-3 entry is 0 because it involves no limit violations; the 2-4 entry is $57.4 = (50.7 - 1.6) + (9.9 - 1.6)$, while 3-4 is $40.8 = (50.7 - 9.9)$

Second Approach: Underestimate

- The gist of the second approach is to pick just one entry from each row and just one from each column that minimizes the sum of the limit violation costs.

Directions: Limit Violation Costs

	2-3	2-4	3-4
Eqv Line 2-3	0	57.4	40.8
Eqv Line 2-4	13.9	0	3.3
Eqv Line 3-4	0	16.2	9.6

This is the Assignment Problem, which can be stated as the optimal assignment of n tasks to n people. The Assignment Problem was solved in the late 1950's using what is known as the Hungarian Algorithm (also known as Munkres' Assignment Algorithm).

Second Approach: Underestimate

Directions: Limit Violation Costs

	2-3	2-4	3-4
Eqv Line 2-3	**0**	57.4	40.8
Eqv Line 2-4	13.9	**0**	3.3
Eqv Line 3-4	0	16.2	**9.6**

For the second approach the new limits would be 50.7 MW for the line between 2-3, 13.8 MW for 2-4 and 9.5 MW for line 3-4.
This is compared with 50.7, 13.8 and 19.1 for the first approach.

Either the high or low limits could be used, and then continue on to the next iteration

Alternative Algorithm

- For most buses a solution exists, so there is no need for a new algorithm

- But for the buses without a solution, just bracketing the limits could eventually result in wide limit ranges.

- Hungarian algorithm is discrete: the selected limits are one of the matrix entries

- Need determination of the "optimal" limit for the line

Quadratic Programming Approach

- Idea is to use quadratic program (QP) to determine the optimal limits that minimizes a cost function, which is the sum of square of normalized TTC mismatches for all directions

- $minimize\left(m^{(2,3)}\right)^2 + \left(m^{(2,4)}\right)^2 + \left(m^{(3,4)}\right)^2$

  – Over estimate: all $m \geq 0$

  – Best estimate: no condition for m

  – Under estimate: all $m \leq 0$

- We find the point with the minimum distance from the origin (all m are zero = exact case) in the feasible region and corresponding line limits

Results for Four Bus System

	Direction	Original Approach		QP	
		Eq. line limit (MW)	Norm. TTC mismatch (%)	Eq. line limit (MW)	Norm. TTC mismatch (%)
Over Estimate	**(2, 3)**	50.8	0.0	50.8	0.0
	(2, 4)	13.8	0.0	13.8	0.0
	(3, 4)	19.2	31.5	19.2	**31.5**
Best Estimate	**(2, 3)**	N/A	N/A	50.8	0.0
	(2, 4)	N/A	N/A	11.7	**-15.2**
	(3, 4)	N/A	N/A	19.2	**11.5**
Under estimate	**(2, 3)**	50.8	-50.4	50.8	0.0
	(2, 4)	13.8	0.0	10.5	**-24.0**
	(3, 4)	9.5	0.0	19.2	0.0

118 Bus Example
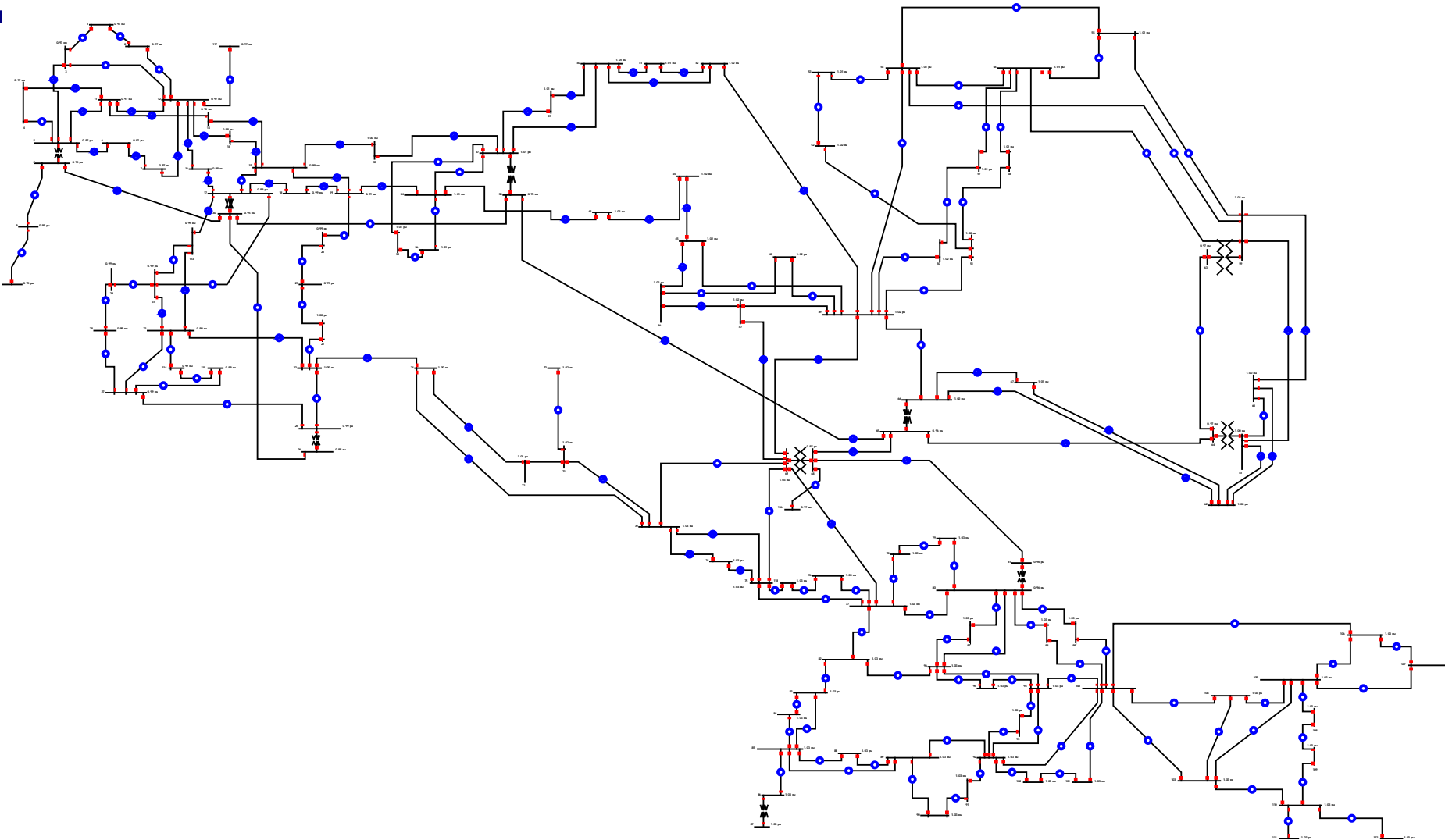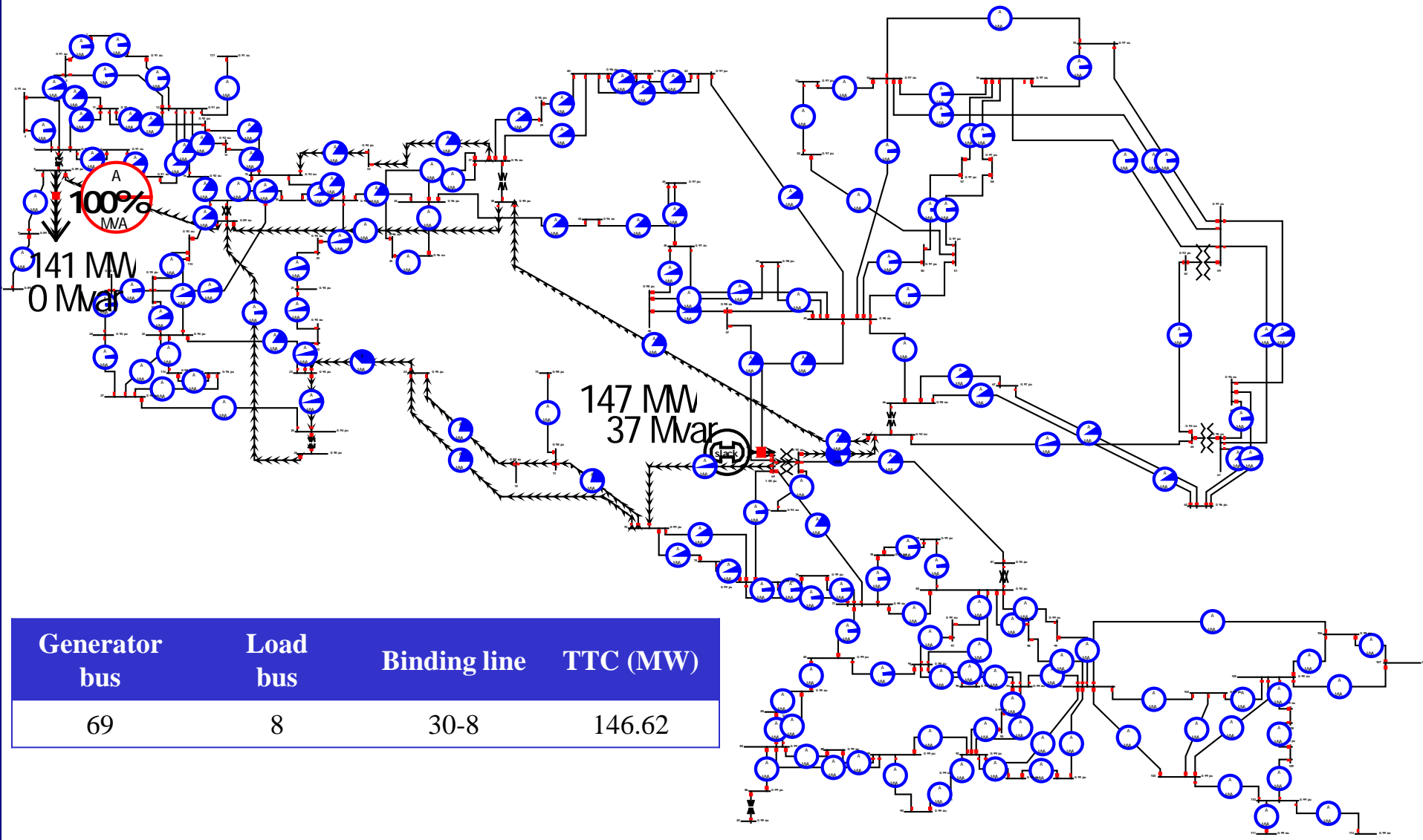
Reduced to a 30 Bus Equivalent

Black lines are retained lines, green lines are equivalent lines, now with limits
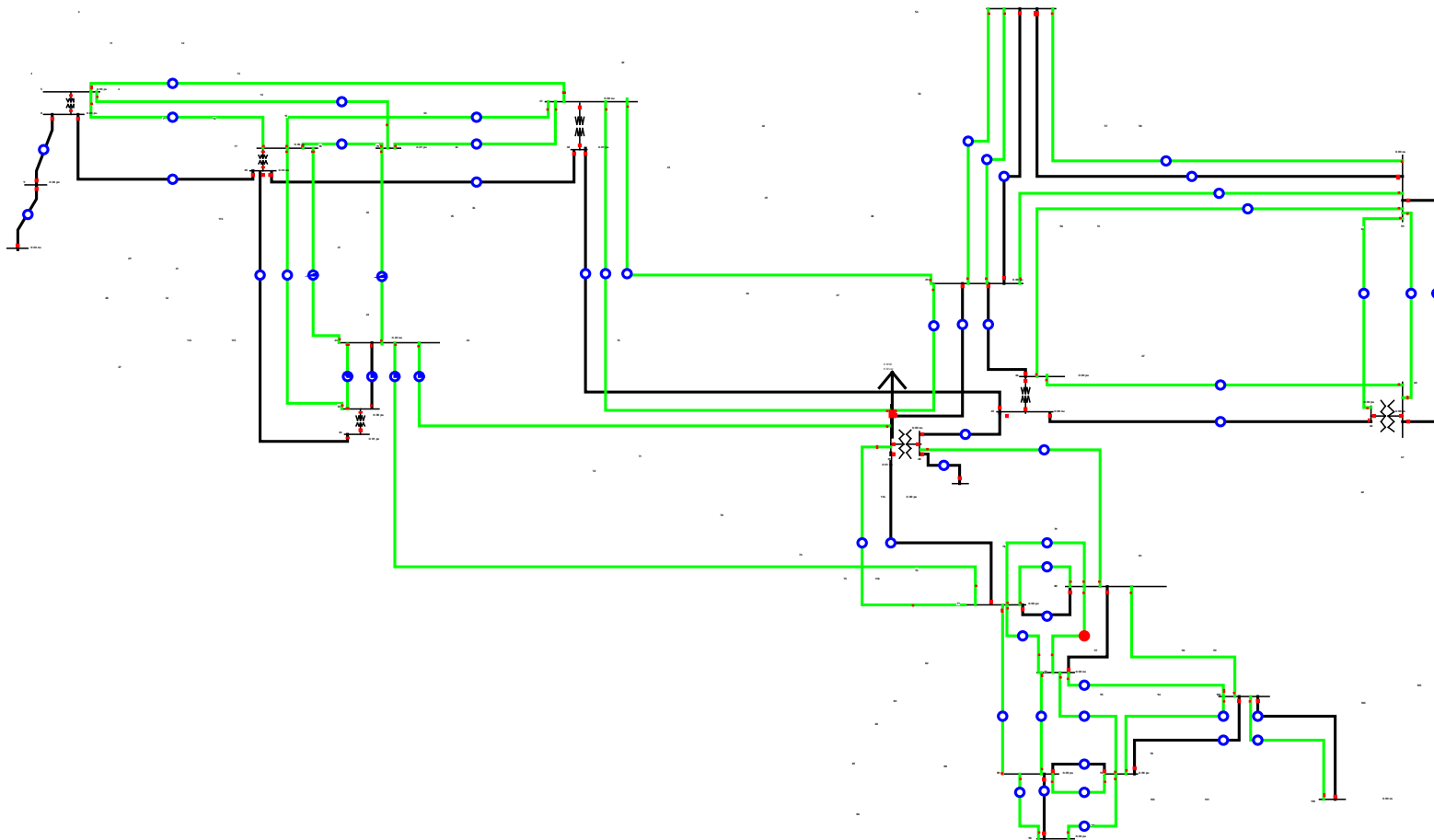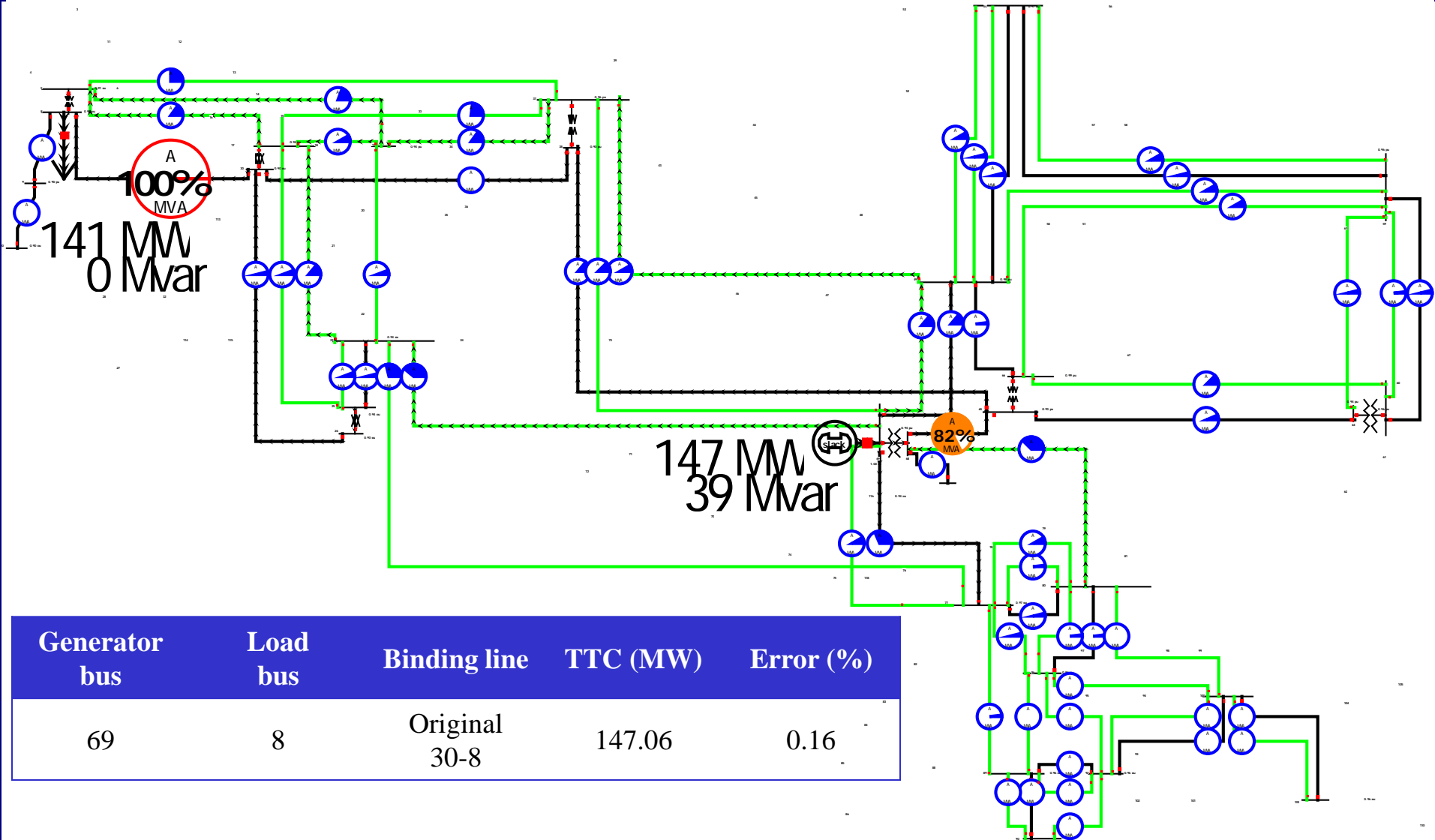
Criteria for Algorithm

- Criteria for limit preserving equivalents
  - Total transfer capability (TTC) of the reduced system matches that of the full system
- Verification
  - Comparison of TTC between a pair of buses that are distant, at least more than one bus in between, in the equivalent system and that of the same buses in the original system
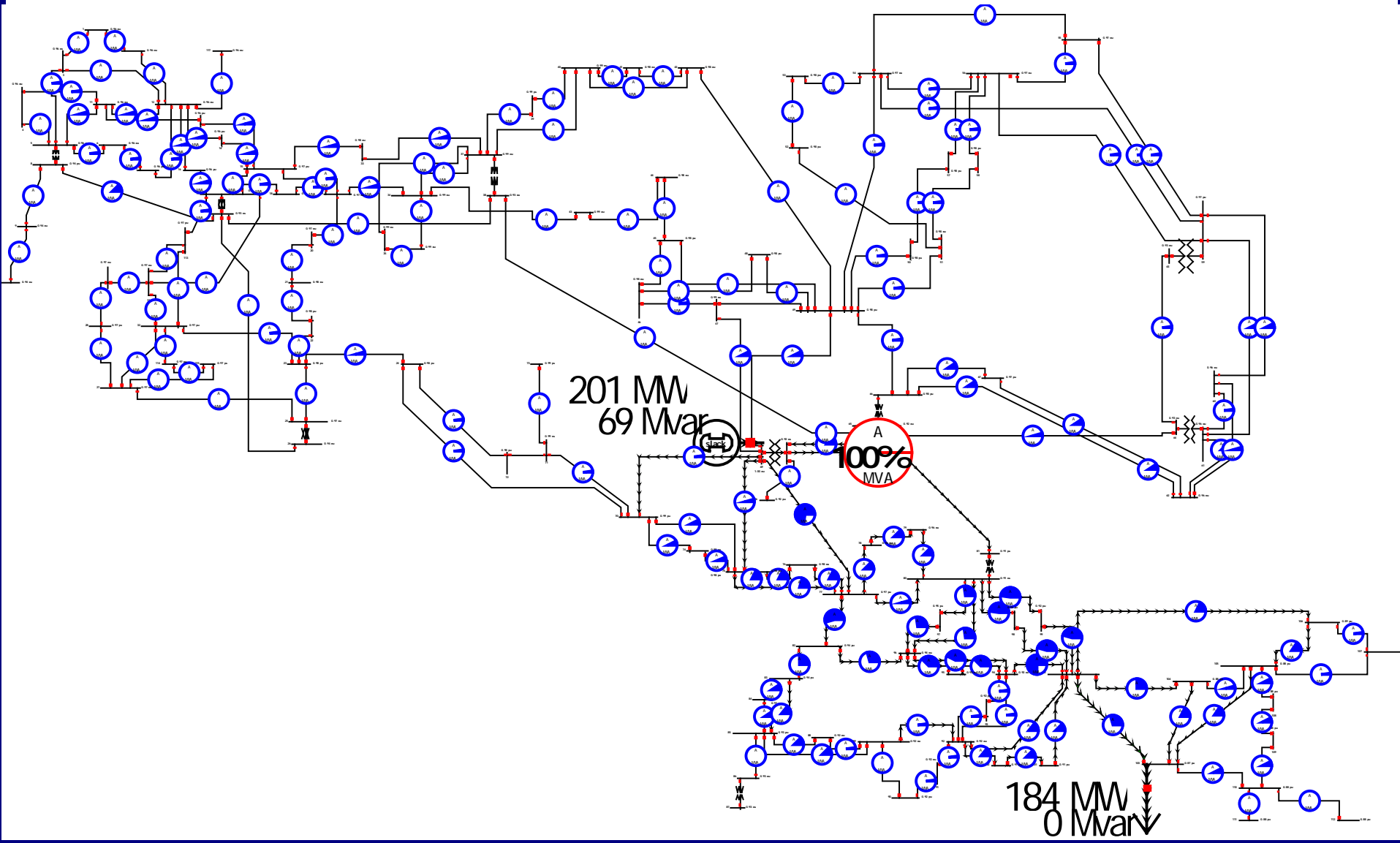
Verification - Example 1

Generator bus	Load bus	Binding line	TTC (MW)
69	8	30-8	146.62

Verification - Example 1

141 MW
0 Mvar

147 MW
39 Mvar

Generator bus	Load bus	Binding line	TTC (MW)	Error (%)
69	8	Original 30-8	147.06	0.16

201 MW
69 Mvar

A
100%
MVA

184 MW
0 Mvar

Verification - Example 2

234 MW
105 Mvar

100%
MVA

100%
MVA

89%
MVA

208 MW
0 Mvar

Generator bus	Load bus	Binding line	TTC (MW)	Error (%)
69	103	Equivalent 68-80	234.0	16.52
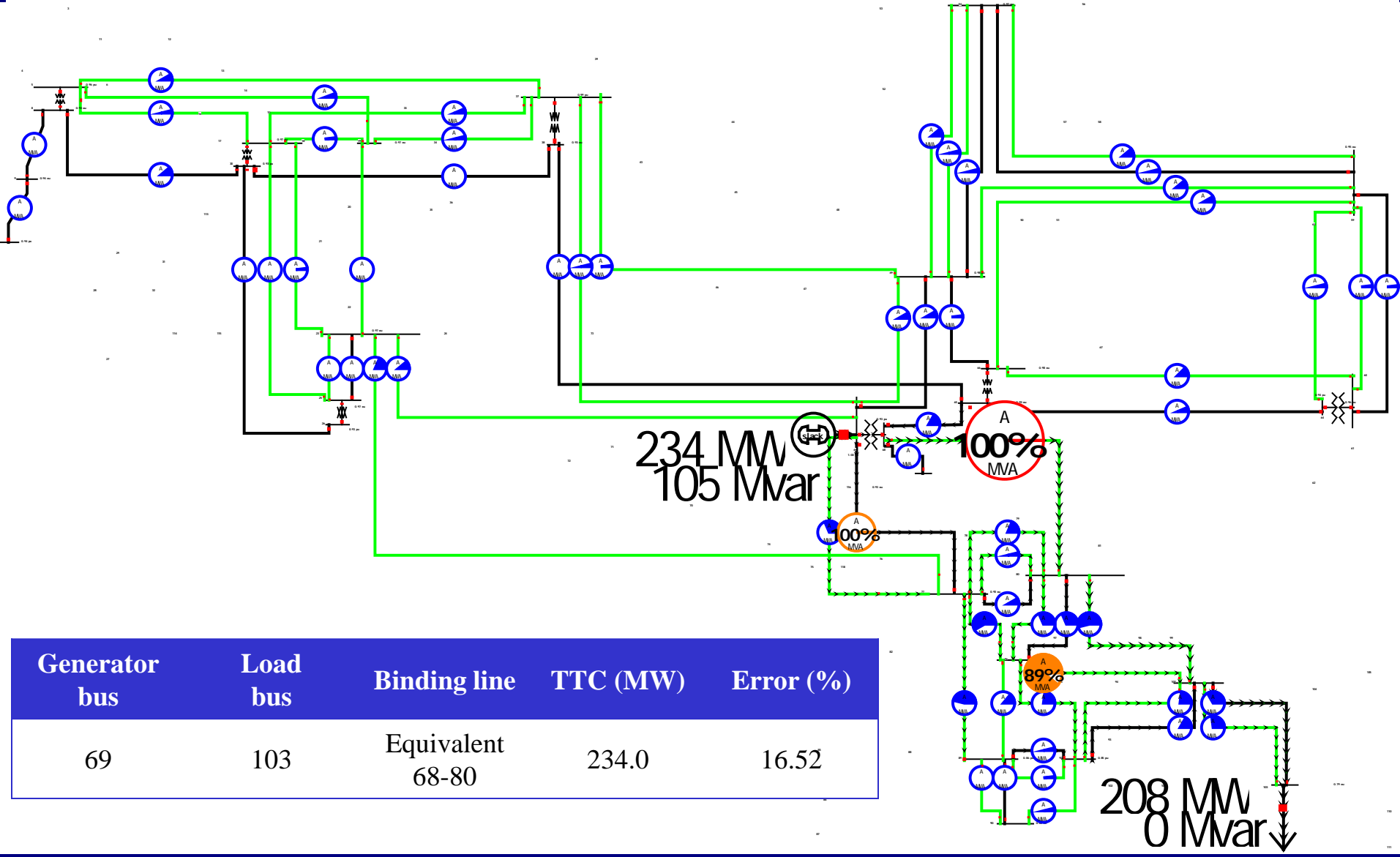
Computational Aspects

- Assume an n bus system, in which m buses are being reduced. Let F_i be the number of first neighbor buses for bus i (a number that will vary during the simulation). Algorithm will be applied sequentially at m buses. For each step we must
  - Calculate $(F_i)^2/2$ PTDFs
  - With sparse vector methods each PTDF has computational order equivalent to the depth of the factorization path, close to $\ln(n)$
- Overall we expect this to be computationally tractable even for large systems, on the order of m $(F_i)^2 \ln(n)$

Computational Aspects, Cont.

- An area of concern is the growth in the first neighbor buses as the system is being equivalenced. However, it has long been recognized that many of these new equivalent lines have quite high impedance and hence they are ignored
  - Limits and hence PTDFs will not need to be calculated for these lines.

Summary and Future Work

- Reduced computational time, perhaps through the use of heuristics for minimizing the number of directions

- Incorporation of bus injections from gen and load
  - TTC reduces to available transfer capability (ATC) to meet existing transmission commitments
  - Our key concern is to prevent operating point dependence

- Additional testing on larger systems

Recent Publications

- W. Jang, S. Mohapatra, T.J. Overbye, H. Zhu, "Line Limit Preserving Power System Network Equivalents," in *Proc. 2013 IEEE Power and Energy Conference at Illinois (PECI)*, Champaign IL, 22-23 Feb. 2013.

- S. Mohapatra, W. Jang and T. J. Overbye, "Equivalent Line Limit Calculation for Power System Equivalent Networks," *IEEE Trans. Power Systems*, accepted in January 2014 for publication.

Questions?